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Automated Block Decomposition of Solid Models Based on Sheet Operations

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Abstract

High quality hex mesh can be effectively generated based on the block structure of a solid model, but the existing approaches to block decomposition of the solid model are still far from the expectation. In this paper, we propose a new method of decomposing the solid model into the block structure based on sheet operations, which can guarantee the validity of the resultant block structure. The method firstly generates an initial hex mesh of the given solid model based on its tet mesh. Then, it inserts a number of global sheets on the boundary and inside the volume respectively by sheet inflation to make the hex mesh capture the geometry of the solid model. Finally our method determines the optimal sheet set that can be extracted based on quality prediction and extracts the sheet set as a whole to get the reasonable and valid block structure of the solid model. Experimental results show the effectiveness of the proposed method.

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Keywords: block decomposition; hex mesh; mesh generation; sheet operation; topological quality; geometric validity.

1. Introduction

In finite element numerical computation, hex meshes are preferred than tet meshes for many reasons [1]: higher computational accuracy; faster convergent speed; smaller storage space. Therefore, special attention is focused on developing automatic algorithms to generate hex meshes during recent decades [2,3]. These hex mesh generation algorithms can be classified into two categories: direct methods and indirect methods. Direct methods generate the hex mesh directly from the solid model or boundary quadrilateral mesh, and this kind of method mainly includes grid-based methods, volume decomposition based methods and plastering methods. Unlike direct methods, indirect methods generate the hex mesh based on the obtained tet mesh, which mainly include H-morph methods. However, because of the strong restrictions of the structure of the hex mesh, automatic and high quality hex meshing for arbitrary shape is still a challenging problem.

It is recognized that volume decomposition methods are promising [3]. The basic idea of this kind of method is to decompose an arbitrary geometry into simple volumes suitable for automatic hex meshing. Among volume

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decomposition methods, block decomposition methods are most ideal since the obtained block structures can be directly used to generate conformal hex mesh. Therefore, in recent years, block decomposition of the solid model receives increasing research attention.

Block decomposition based on 3D frame field has been investigated for several years. Frame is a 3D extension of cross, and it contains three unit vectors which are perpendicular to each other. Nieser et al. [4] propose a method called CubeCover to generate the hex mesh using the calculated frame field information. They are the first researchers to use frame field to generate the hex mesh. Further research is conducted to improve the generation of the initial field. Kowalski et al. [5] build the boundary frames first, and then set up the inner frames according to the smoothness of nodes to obtain the initial field. After optimizing the field by global smoothing, they extract singularity graph based on the field and generate the block structure according to the extracted singularity graph. The frame field based methods calculate the orientations of boundary and inside of the solid model, and fully utilize the information to generate the block structure. These methods can generally generate desirable block structures for many models. However, the existing methods cannot always obtain the correct singularity graphs of complex solid model, and thus cannot guarantee the validity of the resultant structure.

Roca and Sarrate [6,7] propose the Local Dual Contribution algorithm to generate a valid block decomposition based on an initial dual configuration. The algorithm firstly generates a coarse tet mesh, then inserts dual surfaces based on the local dual contribution of tet elements, and lastly generates layers which capture the boundary geometric surfaces in order to obtain the final block structure. The method represents the desired dual configurations through adding local dual contributions, and can efficiently obtain the block structure. However, the quality of the resultant block structure mainly depends on the tet mesh. Kowalski et al. [8] propose a novel approach to the block structure generation based on fun sheet matching. They firstly generate the hex mesh by tet-to-hex method, then insert fundamental sheets into the mesh, and finally generate the block structure by extracting all non-fundamental sheets from the mesh. The approach provides a new way to generate the block structure, which could be very promising. The issue with the approach is that it is not clear whether the hex mesh with only fundamental sheets can well capture the essential geometry of the original model.

Many other methods of volume decomposition are also proposed. Swept volume decomposition approach [9] automatically decomposes solid model into sweepable sub-volumes based on sweep directions extraction, but not all solid models can be fully decomposed into sweepable volumes. Volume decomposition method based on the medial surface [10,11] subdivides solid objects into topologically simple subregions according to the extracted medial surface. This method can generate high quality hex mesh by putting the subdividing cuts between parts with proximate geometry, but generation of steady medial surface itself is still an open problem. Feature recognition can also be used to guide volume decomposition [12]. In general, all these methods do not aim at decomposing solid models into block structures, and thus the conformability between the meshes of adjacent sub-volumes cannot be effectively guaranteed.

It can be observed that a reasonable block structure of a solid model should be the most simplified hex mesh that can capture the geometry of the solid model and has optimal topological quality. However, existing block decomposition methods cannot guarantee to obtain the reasonable block structure of a solid model. Some of them may generate the block which is not a topologically valid hex, and some of them cannot ensure that the obtained block structure completely capture the geometry of the solid model.

In order to effectively generate the reasonable block structure of a solid model, a block decomposition method based on sheet operations is presented in this paper. Its objective is to automatically generate the block structure of a solid model satisfying both topological validity and geometric validity as well as being of good topological quality. The topological validity of a block structure is that every block in the block structure is a topologically valid hexahedron; the geometric validity of a block structure is that the block structure completely captures the geometry of the solid model. To guarantee the topological validity of the block structure, we generate the block structure by conducting sheet operations on an initial hex mesh of the solid model. To be able to effectively generate the initial hex mesh of an arbitrary solid model, we adopt the method that first generates the tet mesh of the solid model, and then converts the tet mesh into a hex mesh by splitting every tetrahedron into four hexahedra. To ensure the geometric validity of the block structure, we globalize the structure of the initial hex mesh by inserting necessary global sheets in the mesh. To make the block structure have good topological quality, we choose the sheet set to be extracted based on the prediction of the quality impact of sheet set extraction.

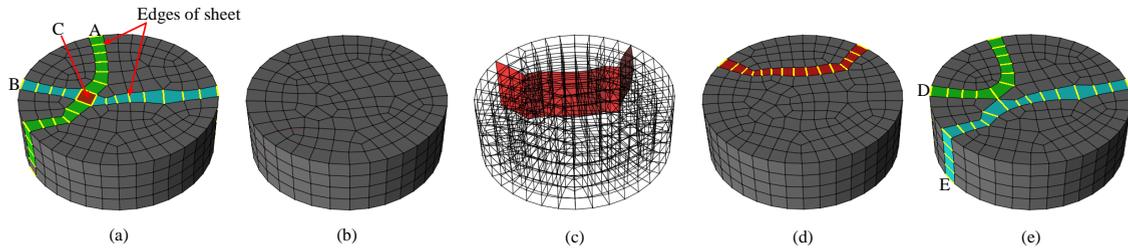


Fig. 1. Sheet operations: (a) sheets A and B, chord C; (b) mesh after extracting sheet A; (c) a quad set; (d) mesh after inflating a sheet along the quad set in (c); (e) mesh after collapsing chord C.

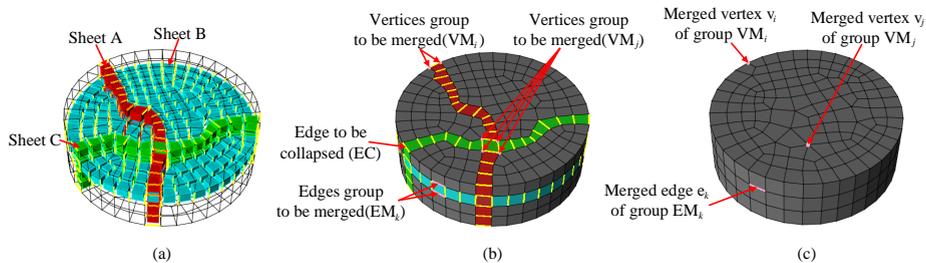


Fig. 2. Illustration of sheet set extraction: (a) the three sheets to be extracted; (b) VM, EC and EM; (c) the resultant mesh after sheet set extraction.

2. Basic Concepts and Approach Overview

2.1. Basic concepts

In this paper, we bring out a novel block decomposition method. Some related concepts are given in this section. Due to the limited space of this paper, we leave out some common concepts like hex mesh and geometric object etc which can be found in [8] [13] [14].

Definition 1 (sheet). Given a hex mesh H and an edge e , a sheet s can be represented as a set of edges $s = E_s$, $E_s \subset H$, $e \in E_s$, where E_s is the maximum set of edges that can be recursively found by getting the topological parallel edge of e .

The basic sheet operations [15] mainly include sheet extraction, sheet inflation and chord collapse, see Fig. 1.

Similar to sheet extraction, sheet set extraction is the operation which together eliminates hexahedra of a set of sheets, and it essentially merges the mesh vertices (VM_i ($i = 1, 2, \dots, m$) is short for each group of vertices to be merged) linked by the edges of this set of sheet into the same mesh vertex v_i ($i = 1, 2, \dots, m$). After sheet set extraction, each VM_i needs to be merged into a single vertex. Several mesh edges need to be collapsed (EC is short for the edges to be collapsed), which is composed of the mesh edges whose start vertices and end vertices locate in the same VM_i ; several mesh edges need to be merged into the same edge (EM_i ($i = 1, 2, \dots, n$) is short for each group of edges to be merged), which is composed of the mesh edges whose start vertices and end vertices respectively locate in the same VM_i .

As illustrated in Fig. 2, after extracting sheets A, B and C, VM_i is merged into v_i , VM_j is merged into v_j , EC is collapsed (colored in yellow), and EM_k is merged into e_k .

Definition 2 (spherical sheet and hemispherical sheet). Given a sheet $s = E_s$, let $\partial E_s \subset E_s$ be the set of boundary edges in E_s , s is called a spherical sheet if $\partial E_s = \emptyset$, that is to say, $\forall e \in E_s$ does not locate on the boundary; s is called a hemispherical sheet if $\partial E_s \neq \emptyset$ and $\forall e \in \partial E_s$ locates on the same geometry surface.

These two types of sheets widely exist in the tet-to-hex mesh whose characteristic is locality, see Fig. 3.

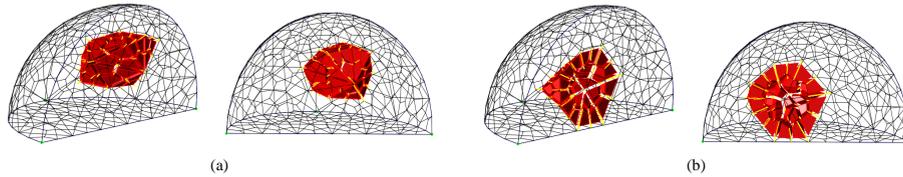


Fig. 3. Two types of local sheets: (a) spherical sheet; (b) hemispherical sheet.

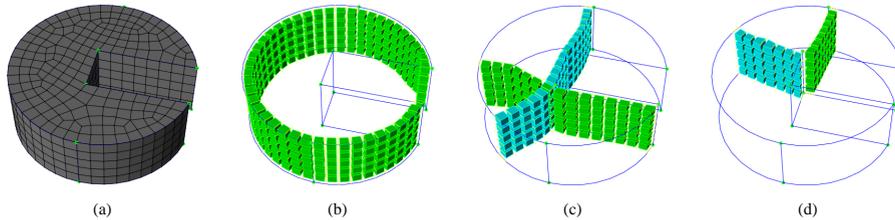


Fig. 4. Fundamental sheets: (a) the hex mesh; (b) level 1 fundamental sheet; (c) level 2 fundamental sheets; (d) level 3 fundamental sheets.

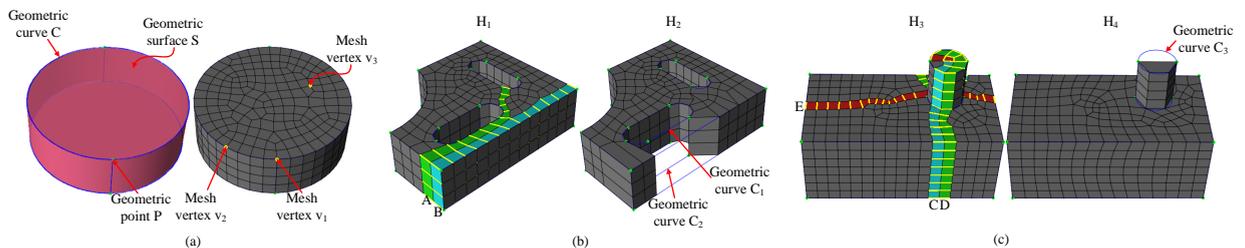


Fig. 5. Illustration of the geometric invalidity of the hex mesh: (a) the geometric attributes of boundary mesh vertices of the hex mesh; (b) the curves C_1 and C_2 are not captured after extracting sheets A and B (the geometric points of the model are colored in green and the geometric curves of the model are colored in blue hereafter); (c) the curve C_3 is not captured after extracting sheets C, D and E.

Definition 3 (Fundamental sheet). Let H be a hex mesh of a B-rep solid model $G = (S, C, P)$, S is the set of geometric surfaces, C is the set of geometric curves, and P is the set of geometric points. A sheet $s = E_s$ is called a fundamental sheet if s captures the geometric surface in S or the geometric curve in C .

Formal definition of fundamental sheet can be found in [8] [13]. According to the difference of associated geometric elements, fundamental sheet can be divided into three categories, as shown in Fig. 4.

Definition 4 (Geometric invalidity of the hex mesh). Given a B-rep solid model $G = (S, C, P)$ and its hex mesh H , H is geometrically valid if the essential geometric information of surfaces in S , curves in C and points in P are captured by H , otherwise H is geometrically invalid.

The specific implication of “capture” can be found in [8] [13]. For the hex mesh which is geometrically valid, all boundary mesh vertices should have their relevant geometric attributes. In other words, each boundary mesh vertex should be attached to a corresponding point of the solid model. As illustrated in Fig. 5(a), vertex v_1 is attached to geometric point P , vertex v_2 is attached to geometric curve C , and vertex v_3 is attached to geometric surface S .

Sheet operations may make the geometrically valid hex mesh invalid. For example, in Fig. 5(b), H_1 is a geometrically valid hex mesh, but after extracting sheets A and B, the resultant mesh H_2 cannot capture the geometric curve C_1 and C_2 . In Fig. 5(c), H_3 is a geometrically valid hex mesh, but after extracting sheets C, D and E, the resultant mesh H_4 cannot capture the geometric curve C_3 . So H_2 and H_4 are geometrically invalid.

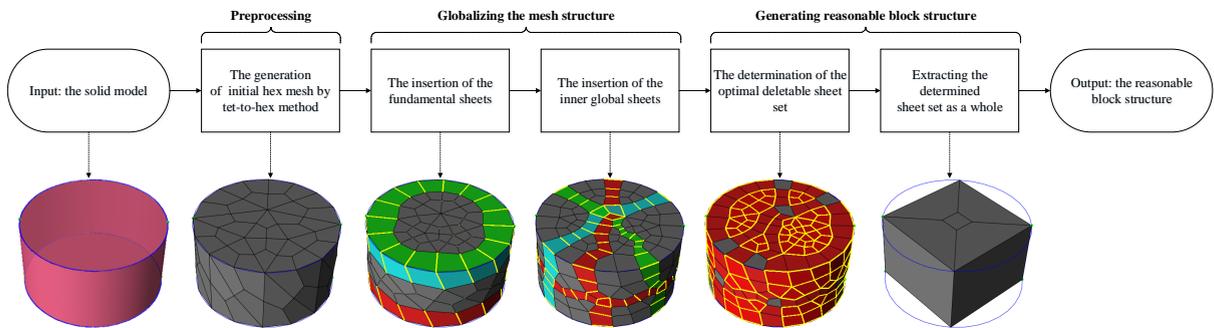


Fig. 6. The flowchart of automated generation of block structure through sheet operations.

2.2. Overview of the approach

In order to achieve automatic generation of high quality hex mesh, an automated block decomposition method based on sheet operations is put forward. The input of our method is a solid model and the output is a reasonable block structure of the solid model that is of topological and geometric validity and has good topological quality. To be able to adopt sheet operations to generate the block structure with topological validity, we first generate an initial hex mesh of the given solid model using central splitting method of tetrahedrons, and then take the initial hex mesh as the major object our method deals with.

For the purpose of effectively generating a reasonable block structure by conducting sheet operations on the initial hex mesh of the solid model, two critical issues need to be solved: 1. how to globalize the structure of the initial hex mesh through sheet operations since most of sheets in the initial hex mesh are local, which is brought about by the generation method of the initial hex mesh; 2. how to obtain the reasonable block structure by performing sheet operations on the globalized hex mesh. For the first issue, we achieve the global hex structure capturing the geometry of the solid model by inserting the necessary global sheets inside and on the boundary of the initial hex mesh through sheet inflation. Regarding the second issue, we first determine all potential deletable sheet sets, then choose the optimal deletable sheet set based on the topological quality prediction, finally obtain the reasonable block structure by extracting the optimal deletable sheet set as a whole.

As illustrated in Fig. 6, our method mainly consists of the following three steps.

Step-1. Preprocessing;

Step-2. Globalizing the mesh structure by sheet inflation;

Step-3. Generating reasonable block structure by sheet set extraction.

3. Preprocessing

For the sake of availability of sheet operations for block decomposition, we generate initial hex mesh of the solid model. In order to ensure the universality of generation of the hex mesh, we adopt central division of tetrahedrons to obtain the initial hex mesh efficiently, as shown in Fig. 7(a).

The hex mesh generated by splitting tetrahedrons has many characteristics, which include :

- **One-to-one correspondence between sheets and original vertices of tet mesh.** Each three hexahedra of a tetrahedron belong to the same sheet, and each tetrahedron is traversed by four sheets, as shown in Fig. 7(b). Therefore, the 2-ring hexahedra of each original vertex of tet mesh compose a single sheet in initial hex mesh. Accordingly, each sheet in initial hex mesh also corresponds to a vertex of the original tet mesh. As illustrated in Fig. 8, the sheets corresponding to the original vertices of tet meshes are colored in red.

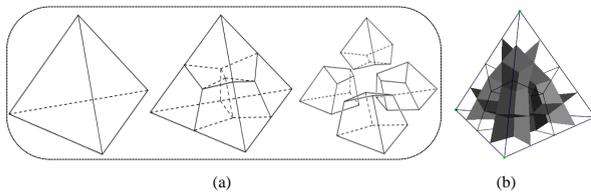


Fig. 7. Tet-to-hex: (a) decomposing a tetrahedron into 4 hexahedra; (b) the 4 local dual sheets of a tetrahedron.

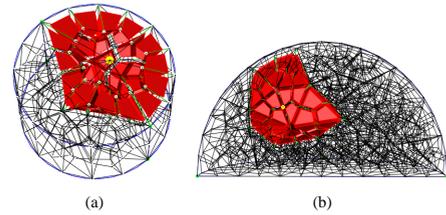


Fig. 8. Some hemispherical and spherical sheets in initial hex meshes: (a) the hemispherical sheet (in red) corresponding to a boundary mesh vertex (in yellow); (b) the spherical sheet (in red) corresponding to an inner mesh vertex (in yellow).

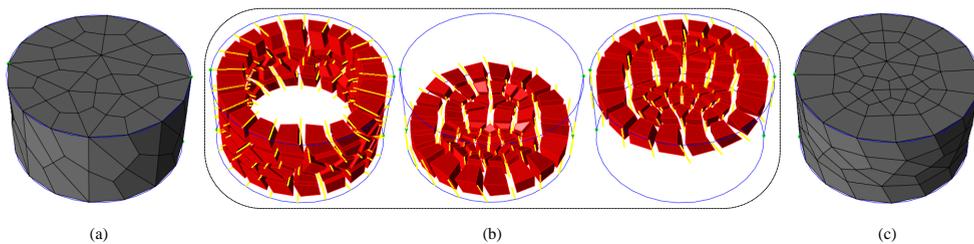


Fig. 9. The insertion of fundamental sheets: (a) the initial hex mesh; (b) the inserted three level 1 fundamental sheets; (c) the resultant fundamental mesh.

- Locality of sheet.** If the original tet vertex locates on geometric surface of the model, then the corresponding sheet is a hemispherical sheet, see Fig. 8(a); if the original tet vertex locates inside the model, then the corresponding sheet is a spherical sheet, see Fig. 8(b). The majority of sheets in the initial tet-to-hex mesh is local.

Based on above observation, it can be seen that the initial hex mesh generated from the tet-to-hex method has a mass of local sheets, and the structure of the mesh is complex. Obviously, the initial mesh does not conform to the requirement of layered structure of high quality hex mesh. Furthermore, it is impossible to get a reasonable block structure by simply coarsening the initial hex mesh.

4. Globalizing the Mesh Structure by Sheet Inflation

The sheets of initial hex mesh cannot capture the global information of the solid model, and in addition, it is difficult to change the locality of such sheets by processing existing sheets. To this end, we insert global sheets on the boundary and inside the model to globalize the hex mesh structure. Numerous global sheets can be inserted but only those which can capture the geometry are inserted. Two kinds of sheets are inserted in our method: the fundamental sheets are inserted firstly and the inner global sheets are inserted secondly.

4.1. The insertion of the fundamental sheets

Fundamental sheet is associated with geometric surface or geometric curve [8]. There, we only insert level 3 fundamental sheet for concave curves. For the cylinder model in Fig. 6, we insert three level 1 fundamental sheets, as shown in Fig. 9. After inserting fundamental sheets, the boundary structure of the hex mesh is globalized, and part of the geometry of the solid model is captured.

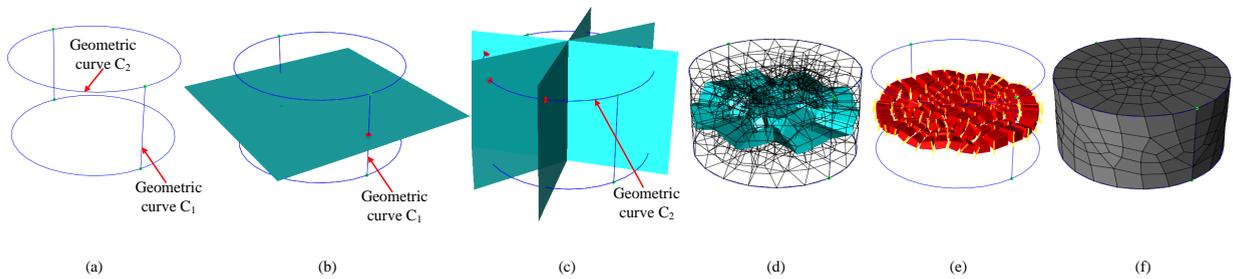


Fig. 10. The insertion of inner global sheets: (a) the cylinder model; (b) the location for the global sheet that can capture the curve C_1 ; (c) the locations for the three global sheets that can capture the curve C_2 ; (d) the determined quad set based on the location in (b); (e) sheet inflation along the quad set in (d); (f) the resultant mesh after inserting 4 inner global sheets.

4.2. The insertion of the inner global sheets

Fundamental sheets can globalize the boundary structure of the hex mesh, and capture geometry of the solid model simultaneously. In addition, we also need to insert inner global sheets to globalize the inner structure of the hex mesh. And in another viewpoint, for many models, fundamental sheets themselves are not sufficient to form valid hex mesh, such as sphere, hence other sheets are demanded to capture remaining geometry. Considering that the fundamental sheets can well capture the geometric surfaces, we insert inner global sheets which can capture geometric curves.

There are four steps to insert global inner sheets, which are listed as follows :

- **Identify all the geometric curves of the model.** For B-rep solid models, they are the geometric curves in boundary representation; for scanned model, they are the identified sharp edges. In this paper, we mainly consider the B-rep solid models. There are 6 curves in the cylinder model as shown in Fig. 10(a).
- **Determine candidate insertion locations of inner global sheets.** Inner sheet is inserted along a plane in order to guarantee the structure of the inserted sheet is global. We process each geometric curve separately: if the curve is straight, we put a plane which passes the midpoint of the curve and is perpendicular to the curve into candidate insertion locations, aiming at capturing geometry of the straight curve, see Fig. 10(b); if the curve is circular arc, we first calculate the central angle α of the circular arc, and then obtain the s section points ($s = \lceil \alpha/45^\circ \rceil - 1$) of the circular arc, and for each section point, put the plane which passes it and is perpendicular to the circular arc into candidate insertion locations, aiming at capturing geometry of the circular arc, see the planes (colored in cyan) in Fig. 10(c); if the curve is a general curve, we put a plane which is perpendicular to the curve at each trisection point into candidate insertion locations, aiming at capturing geometry of the general curve.
- **Select insertion locations of inner global sheets.** We remove same or similar insertion locations which are unnecessary by distance threshold. For the model in Fig. 10(a), 4 insertion locations of inner global sheets are selected finally.
- **Insert inner global sheets.** Insert a global sheet at each of the selected locations by sheet inflation. Every plane of the insertion locations can divide the hex mesh into two parts, and the interface mesh faces between the two parts constitute the quad set for sheet inflation. For example, the quad set in Fig. 10(d) is the mesh faces obtained by the insertion location in Fig. 10(b). We inflate the quad set to generate the inner global sheet, which captures geometric curve. The sheet colored in red in Fig. 10(e) is generated along the quad set in Fig. 10(d).

Based on the above approach of global sheets insertion, we can build inner global sheets to globalize the inner hex mesh and capture all necessary information of geometric curves.

After inserting fundamental sheets and inner global sheets, the structure of the hex mesh is globalized, and whole essential geometry which includes surfaces and curves is preserved.

5. Generating Reasonable Block Structure by Sheet Set Extraction

The hex mesh has been globalized by inserting global sheets. We need to coarsen the mesh to achieve a reasonable block structure.

Due to the strong locality of the sheets in the initial hex mesh, they do not meet the requirement of the globality of block structure, so every sheet in the initial hex mesh (called initial sheet hereafter) should not appear in the final reasonable block structure. Therefore, all these sheets are extracted from the globalized hex mesh, see Fig. 11(b).

Reasonable block structure is the most simplified hex mesh with optimal topological quality, under the premise of ensuring validity. In consideration of this fact, we further simplify the hex mesh by determining the optimal deletable sheet set and uniformly deleting the determined set. In order to ensure the validity of block structure, we rule out the sheet sets which will lead to geometric invalidity of mesh after extracting. In order to improve the efficiency, predictive strategy which pre-estimates the mesh quality after sheet set extraction is adopted.

5.1. Determining the potential deletable sheet sets

Assuming the current hex mesh has n sheets, the number of all possible combinations of sheets is 2^n . We first pre-determine the geometric validity of the hex mesh after extracting each sheet set, and exclude all sets which will result in invalid geometry.

When extracting sheet set, each VM_i needs to be merged into a single mesh vertex v_i ($i = 1, 2, \dots, m$). The geometric attribute of v_i can be determined as follows: if one of the mesh vertices in VM_i is attached to a geometric point P , then v_i is attached to P ; otherwise, if one of the vertices in VM_i is attached to a geometric curve C , then v_i is attached to C ; otherwise, if one of the vertices in VM_i is attached to a geometric surface S , then v_i is attached to S . As the geometric attributes of boundary mesh vertices vary, the change is likely to make the geometry of mesh invalid. Based on the concept of geometric validity introduced in Section 2.1, conditions that sheet set extraction needs to satisfy to guarantee the geometric validity of the resultant mesh are given in Tab. 1.

Table 1. Conditions that sheet set extraction needs to satisfy to guarantee the geometric validity of the resultant mesh.

Conditions	Description
Condition-1	In the case v_i is attached to a geometric point P : any other vertex in VM_i cannot be attached to a geometric point; each geometric curve C that any other vertex in VM_i is attached to should be adjacent to P ; each geometric surface S that any other vertex in VM_i is attached to should be adjacent to P .
Condition-2	In the case v_i is attached to a geometric curve C : any other vertex in VM_i cannot be attached to other geometric curve; each geometric surface S that any other vertex in VM_i is attached to should be adjacent to C .
Condition-3	In the case v_i is attached to a geometric surface S : any other vertex in VM_i cannot be attached to other geometric surface.
Condition-4	There should be at least three mesh vertices attached to the geometric curve which is not straight.

If the hex mesh after extracting the sheet set do not satisfy these conditions, it means extracting this sheet set will make the mesh invalid, thus this sheet set should be removed.

In general, if extracting a sheet set SS will make the hex mesh invalid, then extracting every sheet set which contains SS will make the hex mesh invalid too. Based on the above observation, we can rule out lots of sheet sets which contain the unsuppressible sheet set, thus speed up the determination of the potential deletable sheet sets.

5.2. Predicting the impact on mesh quality of sheet set extraction

For the purpose of determining the optimal sheet set that can be extracted, we predict the impact on mesh quality of extracting each possible sheet set by estimating the impacts on mesh topological quality and mesh density.

5.2.1. Estimating the impact on mesh topological quality

For the hex mesh, topological quality measuring function based on mesh edge valence is more consistent than measuring function based on mesh vertex valence [16]. In this paper, topological quality of the hex mesh is defined

as sum of squares of differences between the current valence and ideal valence of each mesh edge e:

$$T_H = \sum_{\forall e} (v_e - v_{eI})^2 \quad (1)$$

in which v_e is the current valence of mesh edge e, v_{eI} is the ideal valence of mesh edge e. If e locates inside the volume, then $v_{eI} = 4$; if e locates on geometric surface, then $v_{eI} = 3$; if e locates on geometric curve, then $v_{eI} = [(\pi/4 + \alpha)/(\pi/2)] + 1$, $\pi/4 \leq \alpha < 2\pi$, α is the dihedral angle of the geometric curve. A smaller T_H will contribute to a mesh with higher topological quality.

When extracting a sheet set, the mesh edges to be collapsed (EC) are eliminated, and each group of mesh edges to be merged EM_i is merged into a single edge e_i ($i = 1, 2, \dots, n$). As the valences of original mesh edges change, the topological quality of mesh changes too.

The variation of topological quality of mesh after extracting each sheet set (SS for short) can be expressed as :

$$T_{SS} = Q_{EC} + \sum_{i=1}^n Q_{EM_i} \quad (2)$$

in which n is the number of groups of edges to be merged during sheet set extraction. The larger T_{SS} is, the higher the possibility of this sheet set to be extracted is. The detailed description of each criterion is listed below:

- The variation of topological quality of mesh after eliminating EC

$$Q_{EC} = \sum_{\forall e \in EC} (v_e - v_{eI})^2 \quad (3)$$

in which e is the mesh edge in EC. The larger Q_{EC} is, the higher the improvement of topological quality after collapsing EC is.

- The variation of topological quality of mesh after merging each edges group EM_i into edge e_i ($i = 1, 2, \dots, n$)

$$Q_{EM_i} = \sum_{\forall e \in EM_i} (v_e - v_{eI})^2 - (v_{e_i} - v_{e_iI})^2 \quad (4)$$

in which v_{e_iI} is the ideal valence of mesh edge e_i , v_{e_i} is the valence of mesh edge e_i , which will be described in detail. The larger Q_{EM_i} is, the higher the improvement of topological quality after merging EM_i is.

After merging edges group EM_i into edge e_i , the final valence of mesh edge e_i is

$$v_{e_i} = \begin{cases} \sum_{\forall i.e} v_{i.e} - 4(N_I - 1) + \sum_{\forall b.e} v_{b.e} - 3(N_B - 1) - 4 & N_I \geq 1, N_B \geq 1 \\ \sum_{\forall i.e} v_{i.e} - 4(N_I - 1) & N_I \geq 1, N_B = 0 \\ \sum_{\forall b.e} v_{b.e} - 3(N_B - 1) & N_I = 0, N_B \geq 1 \end{cases} \quad (5)$$

in which i.e is the inner mesh edge of EM_i , $v_{i.e}$ is the valence of mesh edge i.e, N_I is the number of inner mesh edges of EM_i , b.e is the boundary mesh edge of EM_i , $v_{b.e}$ is the valence of mesh edge b.e, N_B is the number of boundary mesh edges of EM_i .

5.2.2. Estimating the impact on mesh density

For ease of generating hex mesh of uniform density, blocks of the reasonable block structure should be as uniform as possible. After extracting sheet set, the density of mesh will change. For simplicity, the changes of mesh density on geometric curves are detected to approximate the density change of the whole mesh.

Assuming that the density of initial mesh is uniform, the density of mesh after deleting each sheet set SS can be expressed as:

$$\begin{aligned} V_{SS} &= \text{VAR}(I_{C_iA}) \\ &= \frac{1}{N} \sum_{\forall C_i} \left(I_{C_iA} - \left(\frac{1}{N} \sum_{\forall C_i} I_{C_iA} \right) \right)^2 \end{aligned} \quad (6)$$

in which N is the number of geometric curves C_i of the solid model, $\text{VAR}(\cdot)$ is the variance of density, l_{C_iA} is the average length of mesh edges located on curve C_i , which will be described in detail. A smaller V_{SS} will contribute to a mesh with more uniform density.

After sheet set extraction, the average length of mesh edges in curve C_i is

$$\begin{aligned} l_{C_iA} &= l_{C_i} / (n_f - 1) \\ n_f &= n_o - n_{VM} + n_v \end{aligned} \quad (7)$$

in which l_{C_i} is the length of geometric curve C_i , n_f is the final number of mesh vertices located on curve C_i after sheet set extraction, n_o is the original number of mesh vertices located on curve C_i , n_{VM} is the number of vertices located on C_i to be merged, n_v is the number of final merged vertices located on C_i .

We normalize the Eq. (6)

$$D_{SS} = \frac{1}{V_{SS} + 1} \quad (8)$$

when $V_{SS} \rightarrow 0$, $D_{SS} \rightarrow 1$, it indicates the mesh density will be more uniform after extracting SS.

In order to accelerate the speed of quality prediction, after calculating the mesh edges to be collapsed (EC) and the groups of mesh edges to be merged (EM) of one single sheet, we adopt the recursive strategy $f(n) = f(n - 1) + f(1)$ to calculate EC and EM of n sheets to be extracted, speeding up the estimation of the impact on mesh quality after extracting sheet set.

5.3. Determining and uniformly extracting the optimal deletable sheet set

Considering the reasonable block structure as the most simplified uniform hex mesh with optimal topological quality, we can define the optimal sheet set as SS with highest topological quality T_{SS} , under the premise that density quality D_{SS} is greater than the given threshold λ (in this paper, $\lambda = 0.6$) after sheet set extraction:

$$\begin{aligned} &\underset{SS}{\operatorname{argmax}} T_{SS} \\ &s.t. \quad D_{SS} > \lambda \end{aligned} \quad (9)$$

By extracting the optimal sheet set as a whole, we achieve the reasonable block structure of the solid model. As illustrated in Fig. 11(c), the determined optimal sheet set is colored in red, and the block structure in Fig. 11(d) is obtained by extracting the determined sheet set in Fig. 11(c). Obviously, it is a reasonable block structure of the cylinder model.

In addition, for complex model, the number of the remaining sheets is very large, special strategy is employed to decrease the time complexity of the algorithm. We divide the sheets into different groups and respectively determine the optimal deletable sheet set for each group, and then get the whole deletable sheet set as the union of the above sets. For example, we can firstly choose the optimal deletable sheet set from the inner global sheets, then choose the optimal deletable sheet set from the boundary global sheets, and finally obtain the resultant block structure by extracting the two optimal sheet sets together. This helps to reduce the running time greatly.

Assuming that each sheet set has m edges, the time complexity of quality prediction for each sheet set is $O(m)$. Let the number of the remaining sheets after all the initial sheets are extracted be n and the number of the groups that all the sheets are divided into be k , then the number of all the possible combinations of these sheets is $k * 2^{n/k}$, and the total time complexity of determining the optimal deletable sheet set is $O(m * k * 2^{n/k})$.

6. Experimental Results

For the input B-rep solid models, tet meshes are first generated by the commercial software Abaqus. Then, the proposed method is implemented using C++ as programming language and ACIS as geometric engine. Fig. 12, 13, 14 show the preliminary experimental results, and Tab. 2 illustrates the number of hexahedra(Hex Num.), sheets(Sheet Num.) and singular edges(Singular Num.) of initial hex meshes and resultant block structures, the quality evaluations

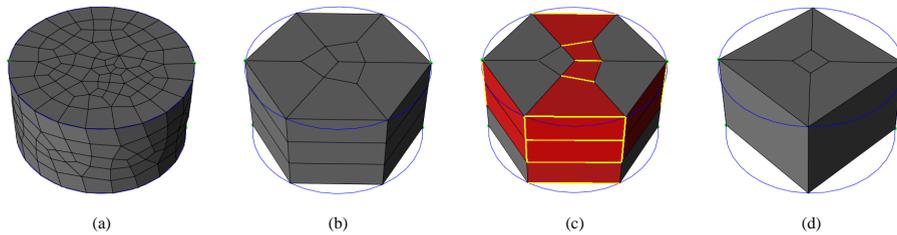


Fig. 11. Generation of reasonable block structure: (a) the hex mesh after globalizing; (b) the hex mesh after extracting initial sheets; (c) the determined optimal deletable sheet set (in red); (d) the final reasonable block structure of the cylinder model after extracting the determined sheet set in (c).

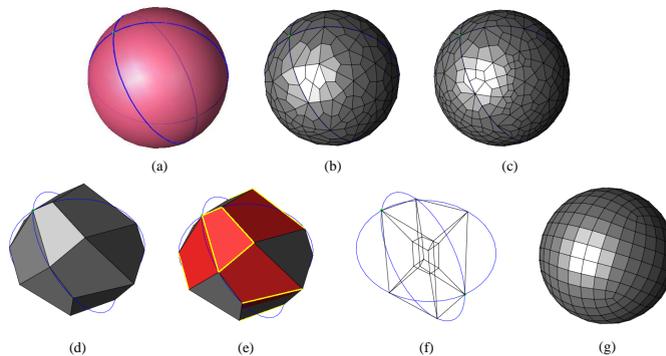


Fig. 12. Automated generation of block structure of Test Case 1: (a) the sphere model; (b) the initial hex mesh; (c) the mesh after inserting global sheets; (d) the mesh after extracting initial sheets; (e) the determined optimal deletable sheet set (in red); (f) the resultant block structure; (g) the final hex mesh generated based on the block structure.

Table 2. The analysis of the results and the running time.

	Hex Num.		Sheet Num.		Singular Num.		Scaled Jacobian (Mean\Min)	Aspect-ratio (Mean\Max)	Hex Skew (Mean\Min)	Running Time(s)			
	Initial	Final	Initial	Final	Initial	Final				Step-1	Step-2	Step-3	Total
Test Case 1	1504	7	105	4	2036	20	0.995\0.970	1.474\2.431	0.988\0.907	0.318	2.971	0.121	3.41
Test Case 2	1952	28	170	13	2492	2	0.983\0.809	1.403\3.933	0.979\0.736	0.404	17.001	20.436	37.85
Test Case 3	16792	69	998	18	24904	18	0.977\0.801	1.878\4.996	0.961\0.722	1.675	129.603	102.146	233.42

including Scaled Jacobian, Aspect-ratio and Hex Skew [17] of the final hex meshes generated based on the block structures, as well as the running time except the generation of tet meshes.

In Fig. 12, we generate the block structure of Test Case 1. The initial hex mesh has 105 sheets, and we insert 1 fundamental sheet and 5 inner global sheets to globalize the hex mesh. Fig. 13 illustrates Test Case 2 with one genus, the initial hex mesh has 170 sheets, and we insert 15 fundamental sheets and 10 inner global sheets. The initial hex mesh of Test Case 3 (Fig. 14) has 988 sheets, and 21 fundamental sheets and 16 inner global sheets are inserted to globalize the mesh. Due to the high density of the initial mesh of Test Case 3, the sheet operations conducted during the block structure generation cost a lot of time.

Comparisons with [8]. The process and result of using the method in [8] to deal with the cylinder model are shown in Fig 15, and ours are shown in Fig 6. It can be seen that our result is better since the resultant block structure obtained by the method in [8] has doublet hexahedra, which is not permissible in many applications. As for the sphere, our result as shown in Fig 12 is of high quality, but the method in [8] cannot generate a valid block structure, because only fundamental sheets are not sufficient to capture the essential geometry of the sphere.

7. Conclusion and Future Work

In this paper, a novel method of automatically generating a reasonable block structure of the solid model for high quality hex meshing is presented. Our method differs from previous approaches in the following two aspects:

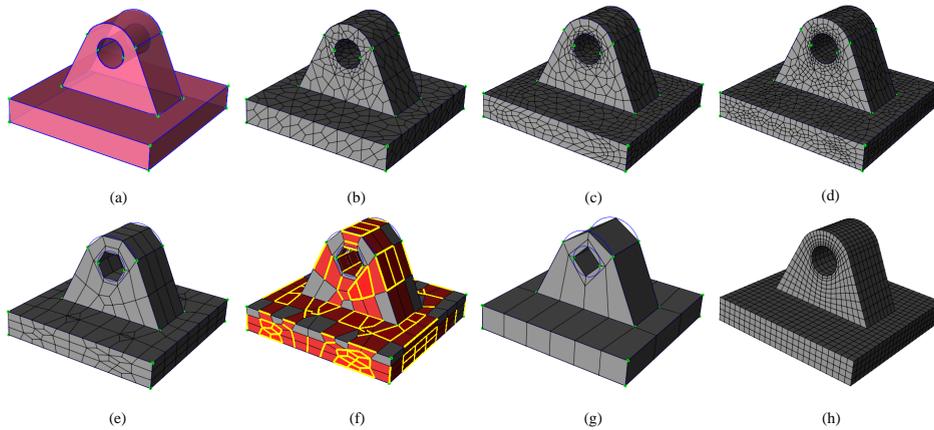


Fig. 13. Automated generation of block structure of Test Case 2: (a) the model with one genus; (b) the initial hex mesh; (c) the mesh after inserting fundamental sheets; (d) the mesh after inserting inner global sheets; (e) the mesh after extracting initial sheets; (f) the determined optimal deletable sheet set (in red); (g) the resultant block structure; (h) the final hex mesh generated based on the block structure.

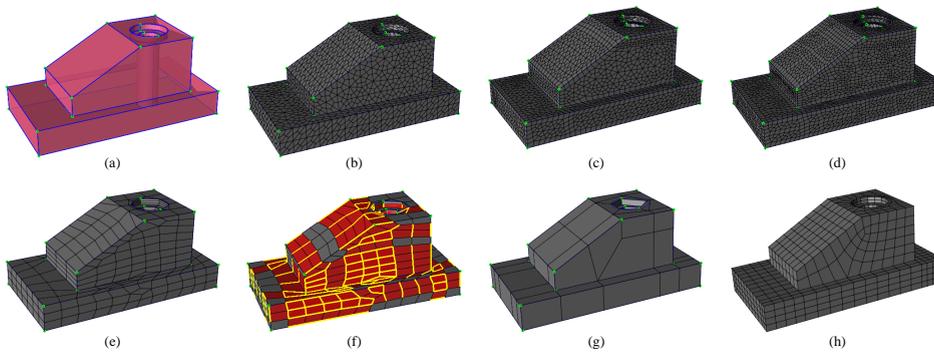


Fig. 14. Automated generation of block structure of Test Case 3: (a) the model with one genus; (b) the initial hex mesh; (c) the mesh after inserting fundamental sheets; (d) the mesh after inserting inner global sheets; (e) the mesh after extracting initial sheets; (f) the determined optimal deletable sheet set (in red); (g) the resultant block structure; (h) the final hex mesh generated based on the block structure.

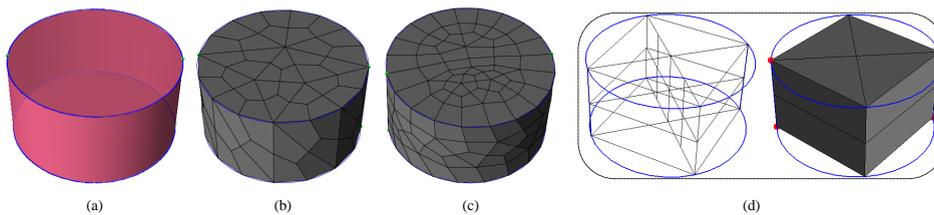


Fig. 15. The process and result of using the method [8] for cylinder model: (a) the cylinder model; (b) the initial hex mesh; (c) the mesh after inserting fundamental sheets; (d) the resultant block structure after extracting all non-fundamental sheets.

- (1) It can guarantee that the generated block structure is of both the topological validity and the geometric validity. The topological validity is achieved by generating the block structure through sheet operations, whereas the geometric validity is realized by globalizing the structure of the initial hex mesh. Compared with [8], we insert not only fundamental sheets, but also the inner global sheets to capture the geometry of the solid model, which aims at guarantee the geometric validity of the resultant block structures.
- (2) It makes the generated block structure have good topological quality by conducting sheet set extraction on the optimal deletable sheet set. The optimal deletable sheet set is determined based on the topological quality predic-

tion and certain heuristics from all possible sheet sets so that the algorithm is effective and efficient. Compared with [8], we determine the optimal deletable sheet set by quality prediction and extract them as a whole, rather than simply extract all non-fundamental sheets one-by-one, being able to ensure the reasonability of block structures.

The current method has several limitations needing to be overcome in our future work:

- (1) In order to obtain the desirable block structure, our current method requires that each curved surface has an appropriate number of boundary curves. For example, we need the sphere surface to have four boundary curves, otherwise the resultant block structure generated by our method will not be optimal. So one of our future work is to solve this issue by automatically generating the required boundary curves for each curved surface.
- (2) The determination of the optimal deletable sheet set is time-consuming, especially for complex solid models. We will study more effective pruning and grouping strategy so as to speed up the determination process of the optimal deletable sheet set in the future.
- (3) Our algorithms of inserting inner global sheets and determining whether sheet set extraction will lead hex mesh geometrically invalid are heuristics based. And because some of heuristic rules are not very general, the block structures generated using them may be not the optimal ones. For example, inserting planar sheet may be not reasonable for many models. We will investigate more general heuristics in the future.

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